Chapter 6 - Day 4

Increasing / Decreasing Function Theorem

- if f is differentiable on an interval I and $f'(x) > 0$ for all points $x \in I$, then f is <u>increasing</u> on I.

- if f is differentiable on an interval I and $f'(x) < 0$ for all points $x \in I$, then f is <u>decreasing</u> on I.

Ex: $f(x) = \dfrac{x+2}{x+3}$. Where is f increasing?

"When is $f'(x) > 0$?"

$$f'(x) = \frac{(1)(x+3) - (x+2)(1)}{(x+3)^2} = \frac{1}{(x+3)^2}$$

$\dfrac{1}{(x+3)^2} > 0$ everywhere except $x = -3$

where $f'(x)$ not defined

$$\boxed{(-\infty, -3) \cup (-3, \infty)}$$

Ex: $f(t) = t^4 - 6t^2 + 7$. Where is $f(t)$ increasing?

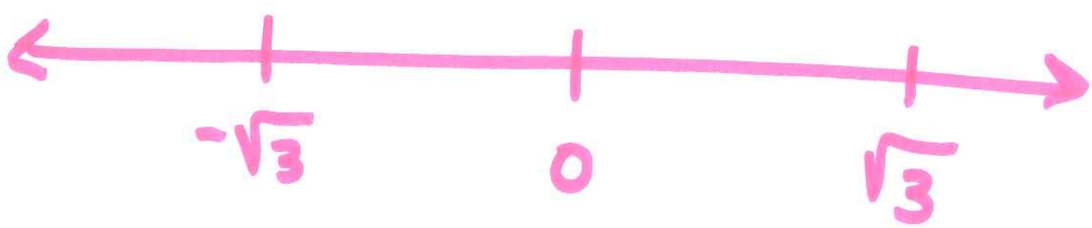
$$f'(t) = 4t^3 - 12t$$
$$= 4t(t^2 - 3) > 0$$

where does it equal 0?

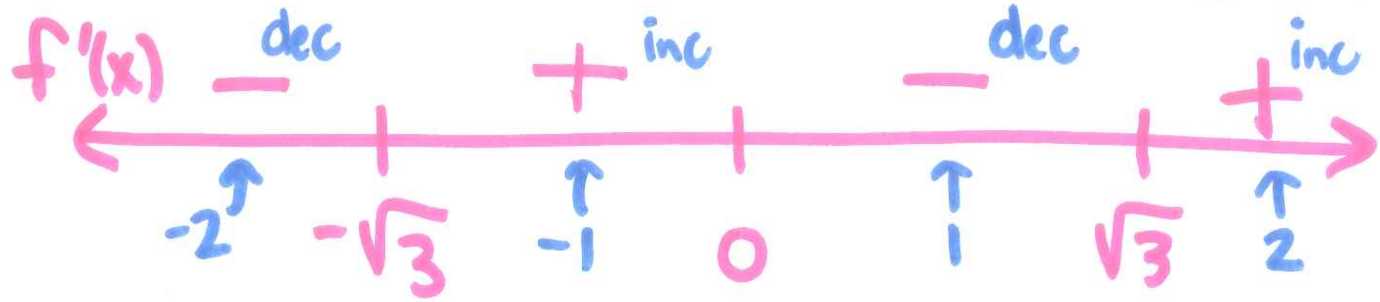
$$4t(t^2 - 3) = 0$$

$$4t = 0 \qquad t^2 - 3 = 0$$
$$\underline{t = 0} \qquad t^2 = 3$$
$$\underline{t = \pm\sqrt{3}}$$

use these to create intervals



Now check a value in each interval to determine where $f'(x)$ positive/negative.



$$f'(-2) = 4(-2)((-2)^2 - 3) = -8 \cdot 1 = -8 \text{ "–"}$$

$$f'(-1) = 4(-1)((-1)^2 - 3) = -4 \cdot -2 = 8 \text{ "+"}$$

$$f'(1) = 4(1)(1^2 - 3) = 4 \cdot -2 = -8 \text{ "–"}$$

$$f'(2) = 4(2)(2^2 - 3) = 8 \cdot 1 = 8 \text{ "+"}$$

$$\boxed{f(t) \text{ increasing } (-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)}$$

Ex: find A such that $h(s) = \frac{1}{(s-7)^2}$ is increasing for all s in the interval $(-\infty, A)$.

$h(s) = (s-7)^{-2}$ thus

$$h'(s) = -2(s-7)^{-3}(1) = \frac{-2}{(s-7)^3}$$

$h'(s) > 0$ when $(s-7)^3 < 0$

$$s-7 < 0$$

$$s < 7$$

$h(s)$ and $h'(s)$ are undefined at $s=7$.

$$h'(0) = \frac{-2}{(0-7)^3} = \frac{-2}{-343} = \frac{2}{343} \text{ "+"}$$

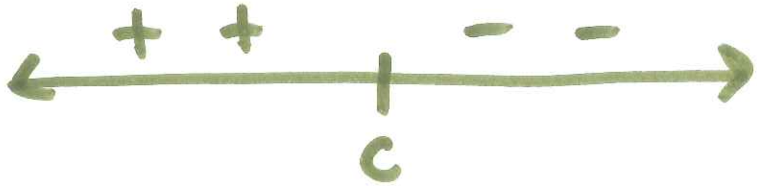
$$h'(8) = \frac{-2}{(8-7)^3} = \frac{-2}{1} = -2 \text{ "−"}$$

$h(s)$ increasing on $(-\infty, 7)$ thus $\boxed{A=7}$.

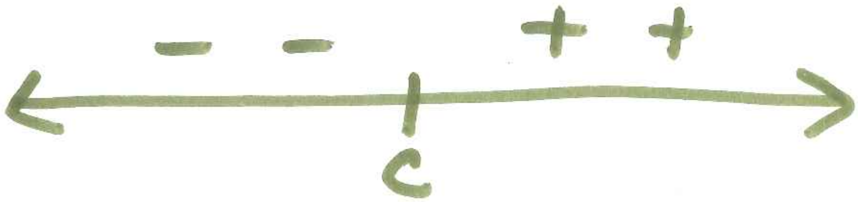
First Derivative Test for Local Max/Min

if f has a critical value at $x=c$ then

- f has a local max at $x=c$ if the sign of f' around c is



- f has a local min at $x=c$ if the sign of f' around c is



Ex: let $g(x) = 3(x-4)^3 - 162$.

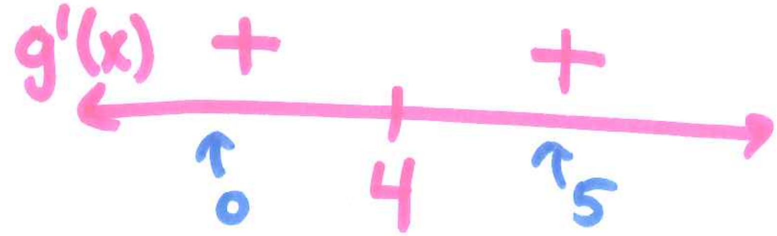
Find the critical points and determine where local max/min are located.

$$g'(x) = 9(x-4)^2 = 0 \text{ when}$$

$$(x-4)^2 = 0$$

$$x - 4 = 0$$

$$x = 4$$



$$g'(0) = 9(0-4)^2 = \text{"}+\text{"}$$

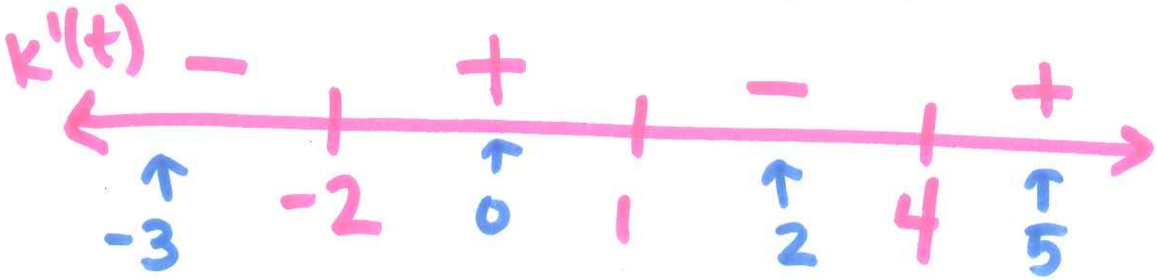
$$g'(5) = 9(5-4)^2 = \text{"}+\text{"}$$

increasing on both sides of $x=4$, so $x=4$ is neither a min or a max.

Ex: $k'(t) = (t-4)(t+2)(t-1)$

Find where $k(t)$ is decreasing.

$k'(t) = 0$ when $t = 4, -2, 1$



$k'(-3) = (-)(-)(-) = -$

$k'(0) = (-)(+)(-) = +$

$k'(2) = (-)(+)(+) = -$

$k'(5) = (+)(+)(+) = +$

$$\boxed{k(t) \text{ decreasing on } (-\infty, -2) \cup (1, 4)}$$

local max at $t = 1$

local min at $t = -2, 4$